

# Selected aspects of Operations Research

Guest lecture at the

**Ural State University of Railway Transport  
Yekaterinburg**  
January 2013

**Prof. Dr. Mike Steglich**  
Technical University of Applied Sciences Wildau

TUAS Wildau / Prof. Dr. M. Steglich

Selected aspects of Operations Research

## 1 Operations Research

- ⇒ **Operations Research** (often referred to as Management Science) is a scientific approach to decision making that seeks to best design and operate a system, usually under conditions requiring the allocation of scarce resources. ... The scientific approach to decision making usually involves the use of one or more mathematical models. [Winston (2003), p. 1.]
- ⇒ **Operations research (OR)** is the development and use of quantitative models and methods for decision making in companies and organizations. [Suhl/Mellouli (2009), p. 1.]
- ⇒ **Characteristics**
  - Preparing the decision making process to aim for optimal decisions -> **mathematical modelling**
  - Use of **mathematical methods**

## 1 Operations Research

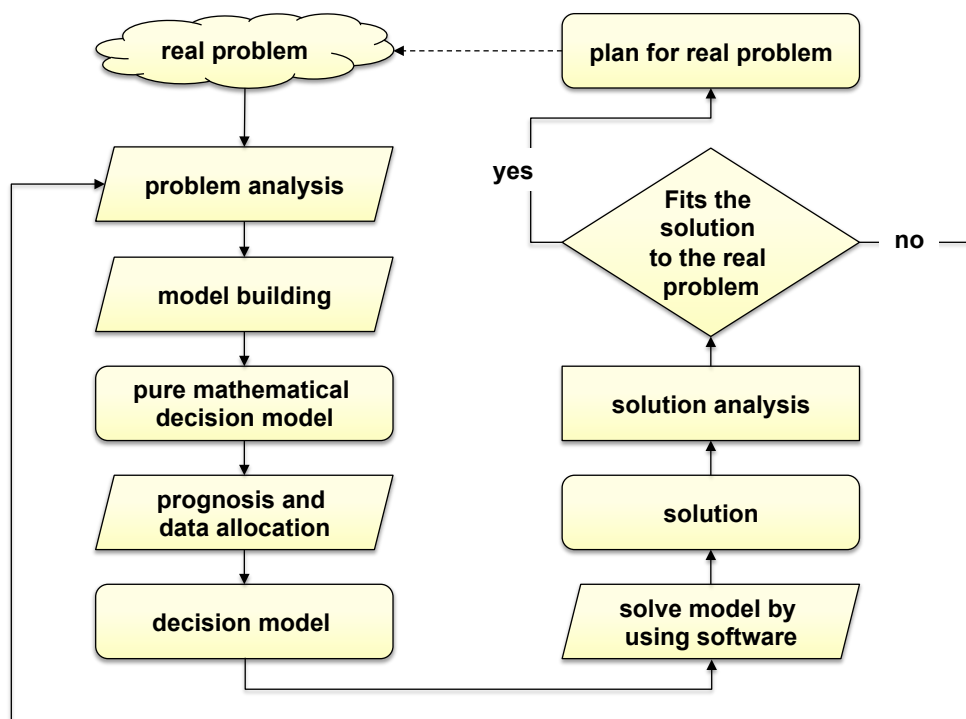
### ⇒ Selected Operations Research Applications [www.hsor.org]

- airlines - scheduling planes and crews, pricing tickets, taking reservations, and planning the size of the fleet
- logistics companies - routing and planning
- financial services - credit scoring, marketing, and internal operations
- lumber and wood products - managing forests and cutting timber
- local government - deployment of emergency services
- policy studies and regulation - environmental pollution, air traffic safety, AIDS, and criminal justice policy

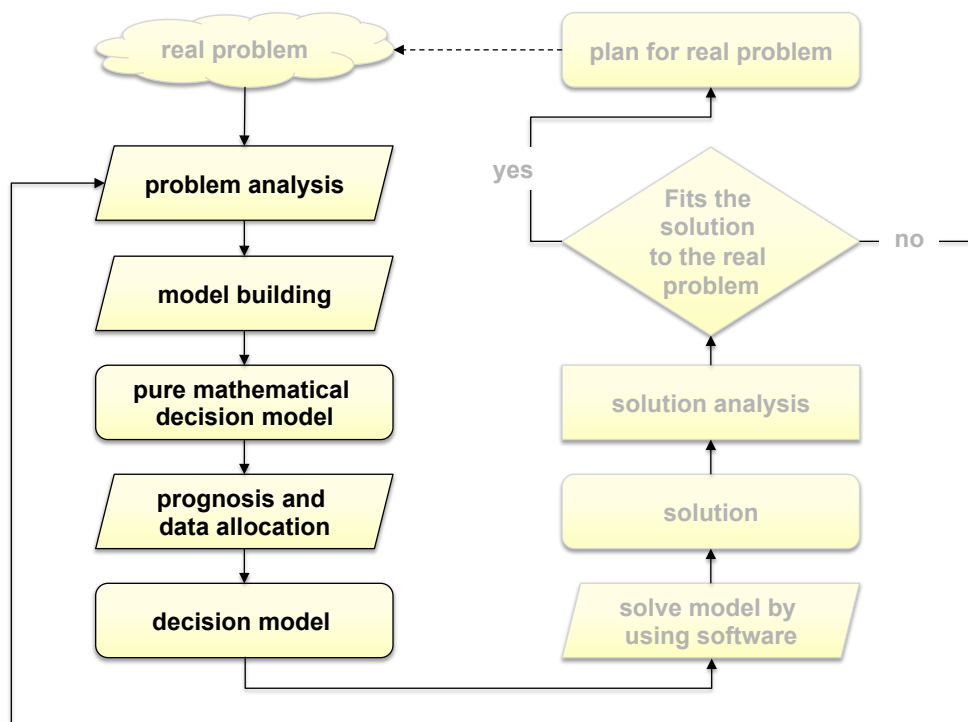
### ⇒ Selected Operational research problem-solving techniques and methods

- decision theory
- optimization
- simulation
- statistics
- probability theory
- queuing theory
- game theory
- graph theory

## 1 Operations Research



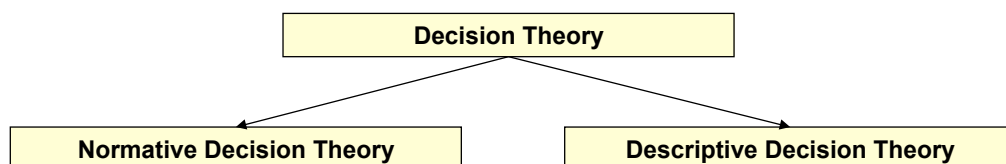
## 2 Decision Making



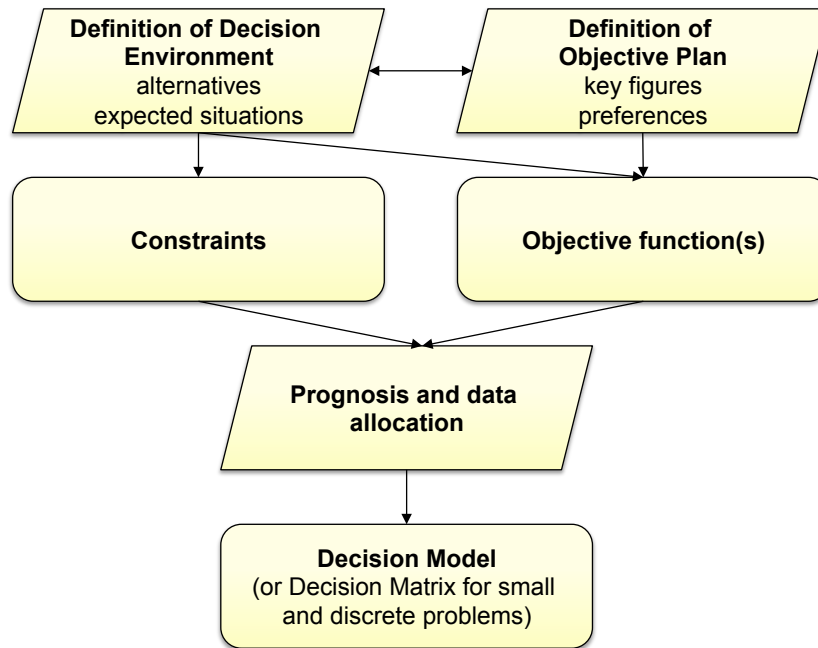
## 2 Decision Making

### ⇒ Decision and Decision Theory

- **Decision:** a choice of one of more possible alternatives
- The **Decision Theory** systematically deals with such action choices or decisions



## 2 Decision Making



## 2 Decision Making

### ⇒ Key Figures

- Definition of all relevant figures (e.g. profit, revenue, cost, ... )

### ⇒ Preferences

- **Value preference**
  - benefit of a figure based on the value (maximize, minimize, approximate)
- **Type preference**
  - benefit of a figure based on the type
- **Risk preference**
  - benefit of a figure because of the likelihood of the results in diverse possible situations
- **Time preference**
  - benefit of a target figure because of the temporal occurrence of the results in a multi-periodic observation period

### ⇒ Alternatives

- amount of all possible alternatives
- principle of completeness and exclusiveness
- finite and endless set of actions
- discrete and continuous set of actions

### ⇒ Expected Situations

- amount of all possible situations with an impact on the alternatives results

## 2 Decision Making

### ⇒ Decision Model (or Optimization Model)

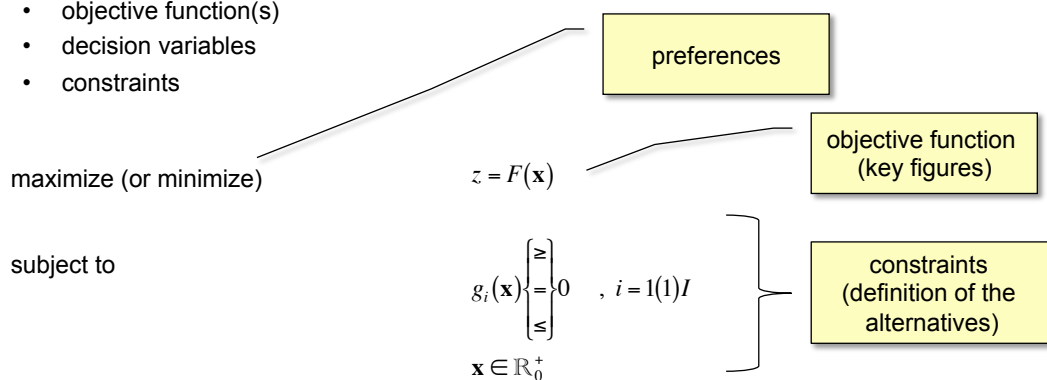
- Combination of all parts of the Decision Environment and the objective plan as a mathematic model.
- This model is to be solved with adequate algorithm.

### ⇒ Decision Matrix (discrete problems)

- The result matrix is successively subordinated to the preferences of the decision maker.
- The decision maker chooses the alternative with the highest benefit.

## 2 Decision Making

- ⇒ An **optimization model** is a mathematical representation of a real decision problem and consists of
- objective function(s)
  - decision variables
  - constraints



$\mathbf{x}$  vector of the decision variables  $x_j; j=1(1)J$   
 $F(\mathbf{x})$  objective function  
 $\mathbf{x} \in \mathbb{R}_0^+$  non-negative constraint

## 2 Decision Making

⇒ A company plans to produce two types of products (P1 and P2).

- The profit contribution per unit of P1 is €10 and for P2 €20.
- Both products require a machine's capacity. One unit of P1 (P2) requires 1 (1) hour of the capacity. Only 100 hours are available.
- A second production factor is material, for which 720 kg is available. Product 1 needs 6 kg per unit and P2 9 kg per unit.
- The demand for P2 is limited to 60 units per period.
- The company wants to maximize the profit contribution and has to determine the optimal production mix.

$$pc = 10 \cdot x_1 + 20 \cdot x_2 \rightarrow \max!$$

s.t.

$$x_1 + x_2 \leq 100$$

$$6 \cdot x_1 + 9 \cdot x_2 \leq 720$$

$$x_2 \leq 60$$

$$x_1, x_2 \geq 0$$

machine

material

upper bound

non-negative restriction

### Objective function

key figure: profit contribution

Value preference: max!

Type preference, Risk preference, Time preference : n/a

### Constraints

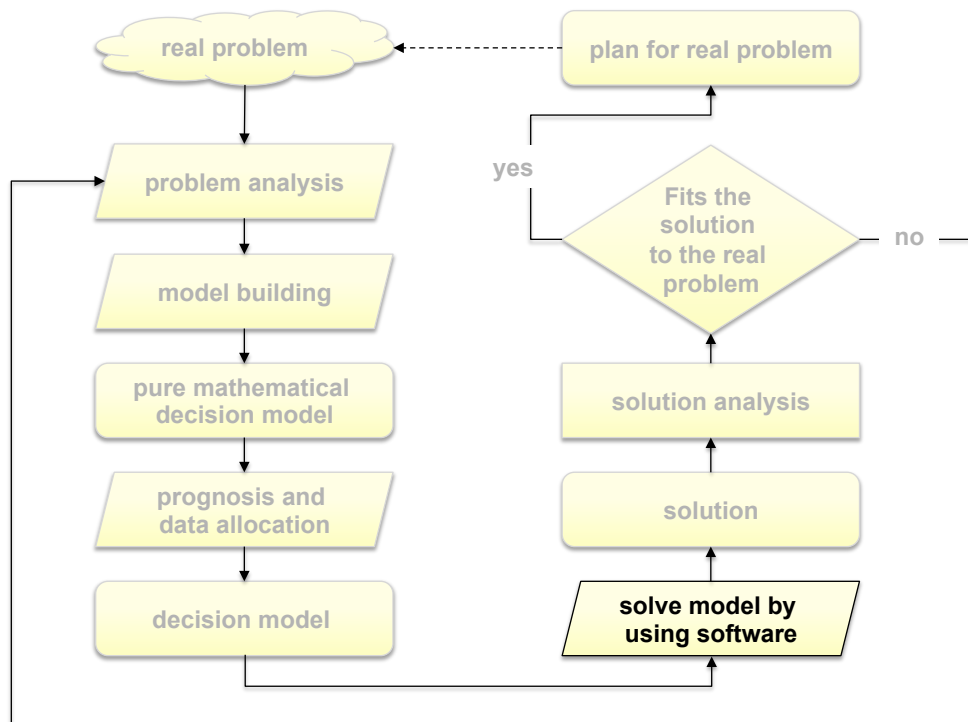
finite and continuous set of actions

## 2 Decision Making

⇒ **Classifications of optimization models**

- Character of the model functions: **linear and nonlinear models**
- Number of relevant possible situations: **deterministic and probabilistic (stochastic) models**
- Number of periods: **static and dynamic models**
- Type of the decision variables: **integer and non-integer models**
- Number of the objectives: **uni-objective and multi-objective models**

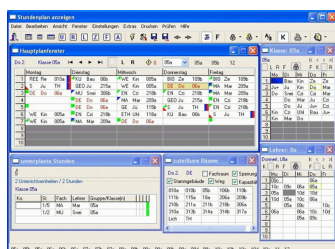
### 3 Optimization Software



### 3 Optimization Software

#### Specialized Optimization Software

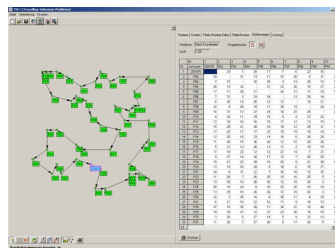
- Software intended to solve a particular decision problem.



<http://www.fuxschool.de/>

#### Problem orientated Optimization Software

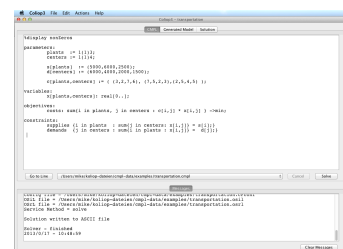
- Software that can be used for a class of decision problems.
- This kind of software works like a template that is to be customized by the decision maker.



[TSP by Dieter Feige]

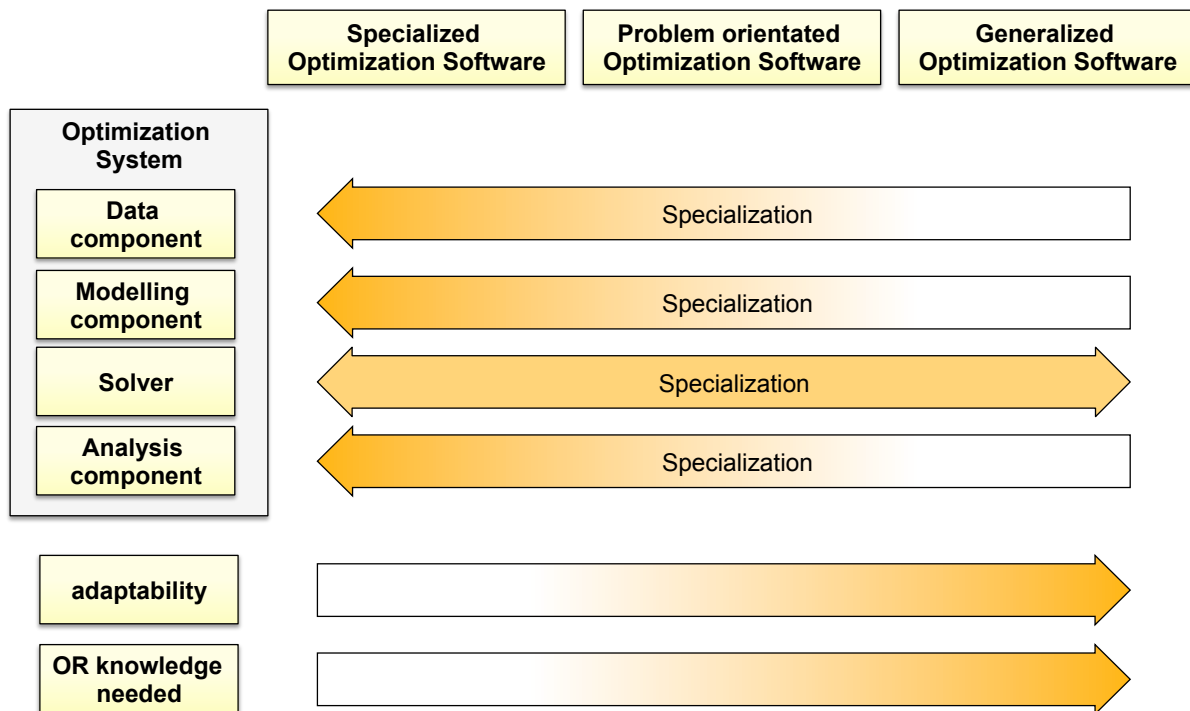
#### Generalized Optimization Software

- Software that can be used for a wide range of decision problems.
- e.g.
  - Spread sheet software combined with solvers
  - Mathematical Programming Languages
  - Algebraic Systems
  - APIs of commercial or Open Source solvers



<http://colliop.org>

### 3 Optimization Software



### 4 CMPL <Coliop|Coin> Mathematical Programming Language

- ⇒ CMPL (<Coliop|Coin> Mathematical Programming Language) is a mathematical programming language and a system for mathematical programming and optimization of linear optimization problems.
- ⇒ CMPL executes the COIN-OR OSSolverService, GLPK, Gurobi, SCIP and CPLEX directly to solve the generated model instance. Since it is also possible to transform the mathematical problem into MPS, Free-MPS or OSiL files, alternative solvers can be used.
- ⇒ CMPL is an open source project licensed under GPLv3. It is written in C++ and is available for most of the relevant operating systems (Windows, OS X and Linux).
- ⇒ CMPL is a COIN-OR project initiated by the Technical University of Applied Sciences Wildau and the Institute for Operations Research and Business Management at the Martin Luther University Halle-Wittenberg.



## 4 CMPL <Coliop|Coin> Mathematical Programming Language

⇒ **Working steps:**

1. Modelling the problem with CMPL
2. Solve the model
3. Solution analysis

$$\begin{aligned}
 pc &= 10 \cdot x_1 + 20 \cdot x_2 \rightarrow \max! \\
 \text{s.t.} \\
 x_1 + x_2 &\leq 100 \\
 6 \cdot x_1 + 9 \cdot x_2 &\leq 720 \\
 x_2 &\leq 60 \\
 x_1, x_2 &\geq 0
 \end{aligned}$$

## 4 CMPL <Coliop|Coin> Mathematical Programming Language

1. Modelling the problem with CMPL

```

parameters:
  products := 1(1)2;
  restrictions := 1(1)3;

  c[products] := (10, 20);
  a[restrictions, products] := ((1, 1), (6, 9), (0,1));
  b[restrictions] := (100, 720, 60);

variables:
  x[products] : real[0..];

objectives:
  c[]T * x[] ->max;

constraints:
  a[, ] * x[] <=b[];

```

## 4 CMPL <Coliop|Coin> Mathematical Programming Language

2. Solve the model
3. Solution analysis

```

-----
Problem                first-example.cmpl
Nr. of variables        2
Nr. of constraints      2
Objective name          line_1
Solver name             COIN-OR clp
Display variables       (all)
Display constraints     (all)
-----

Objective status        optimal
Objective value         1500 (max!)

Variables
Name                    Type          Activity    Lower bound    Upper bound    Marginal
-----
x[1]                    C              30              0             Infinity       0
x[2]                    C              60              0              60             5
-----

Constraints
Name                    Type          Activity    Lower bound    Upper bound    Marginal
-----
line_2                  L              90          -Infinity       100            -
line_3                  L             720          -Infinity       720           1.66667
-----

```

## 5 Transportation problems

- ⇒ A Transportation problem is a special kind of linear programming problem which seeks to minimize the total shipping costs of transporting goods from several supply locations (origins or sources) to several demand locations (destinations).
- ⇒ Assumptions and definitions:
- The goods available at each supply location are limited – supply  $s_i$ ;  $i=1(1)m$ .
  - The demand at each demand location is known – demand  $d_j$ ;  $j=1(1)n$
  - The total demand and the total supply are equal.  $\sum_{i=1}^m s_i = \sum_{j=1}^n d_j$
  - There are costs  $c_{ij}$  if a unit is shipped from source  $i$  to the destination  $j$ .
  - The variables  $x_{ij}$  describe the amount of units that are to be transported from source  $i$  to the destination  $j$ .

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij} \cdot x_{ij} \rightarrow \min!$$

s.t.

$$\sum_{j=1}^n x_{ij} = s_i \quad ; i = 1(1)m$$

$$\sum_{i=1}^m x_{ij} = d_j \quad ; j = 1(1)n$$

$$x_{ij} \geq 0 \quad ; i = 1(1)m, j = 1(1)n$$

## 5 Transportation problems

⇒ Transportation problems faced by Foster Generators.

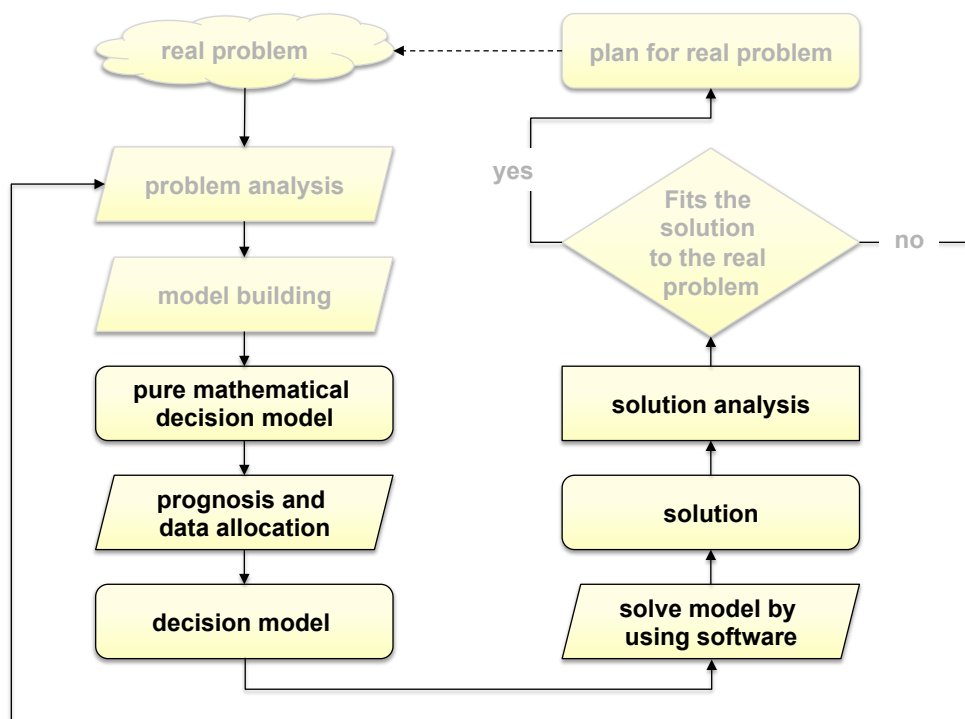
- This problem involves the transportation of a product from three plants to four distribution centers.
- Foster Generators operates plants in Cleveland, Ohio; Bedford, Indiana; and York, Pennsylvania. The supplies are defined by the production capacities over the next three-month planning period for one particular type of generator.
- The firm distributes its generators through four regional distribution centers located in Boston, Chicago, St. Louis, and Lexington.

Origin	Destination				Supply
	Boston	Chicago	St. Louis	Lexington	
Cleveland	3	2	7	6	5000
Bedford	7	5	2	3	6000
York	2	5	4	5	2500
<b>Demand</b>	6000	4000	2000	1500	

costs  $c_{ij}$  if a unit is shipped from plant  $i$  to the distribution center  $j$

- The management has to decide how much of its products should be shipped from each plant to each distribution center. The objective is to minimize the transportation costs.

## 5 Transportation problems



## 5 Transportation problems

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij} \cdot x_{ij} \rightarrow \min!$$

s.t.

$$\sum_{j=1}^n x_{ij} = s_i \quad ; i = 1(1)m$$

$$\sum_{i=1}^m x_{ij} = d_j \quad ; j = 1(1)n$$

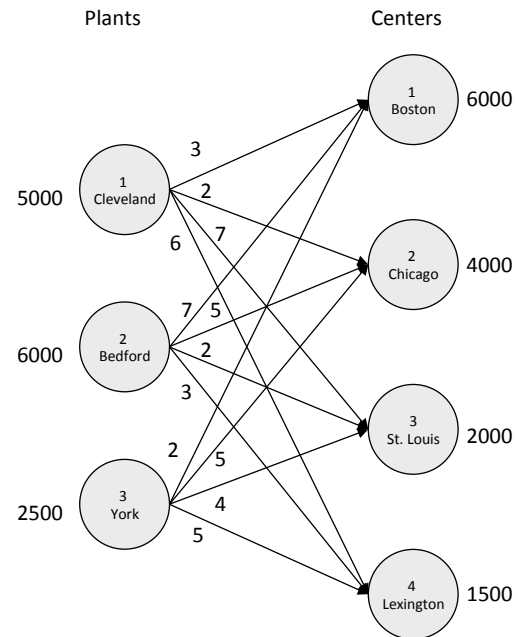
$$x_{ij} \geq 0 \quad ; i = 1(1)m, j = 1(1)n$$

$x_{ij}$  – number of units shipped from plant  $i$  to center  $j$

$c_{ij}$  – cost per unit of shipping from plant  $i$  to center  $j$

$s_i$  – supply in units at plant  $i$

$d_j$  – demand in units at destination  $j$



## 5 Transportation problems

parameters:

```
plants := 1(1)3;
centers := 1..4;
```

```
s[plants] := (5000,6000,2500);
d[centers] := (6000,4000,2000,1500);
```

```
c[plants,centers] := ( (3,2,7,6), (7,5,2,3), (2,5,4,5) );
```

variables:

```
x[plants,centers]: real[0..];
```

objectives:

```
costs: sum{i in plants, j in centers : c[i,j] * x[i,j] } ->min;
```

constraints:

```
supplies {i in plants : sum{j in centers: x[i,j]} = s[i];}
demands {j in centers : sum{i in plants : x[i,j]} = d[j];}
```

$$\sum_{i=1}^m \sum_{j=1}^n c_{ij} \cdot x_{ij} \rightarrow \min!$$

s.t.

$$\sum_{j=1}^n x_{ij} = s_i \quad ; i = 1(1)m$$

$$\sum_{i=1}^m x_{ij} = d_j \quad ; j = 1(1)n$$

$$x_{ij} \geq 0 \quad ; i = 1(1)m, j = 1(1)n$$

## 5 Transportation problems

> `cmp1 transportation.cmpl`

```

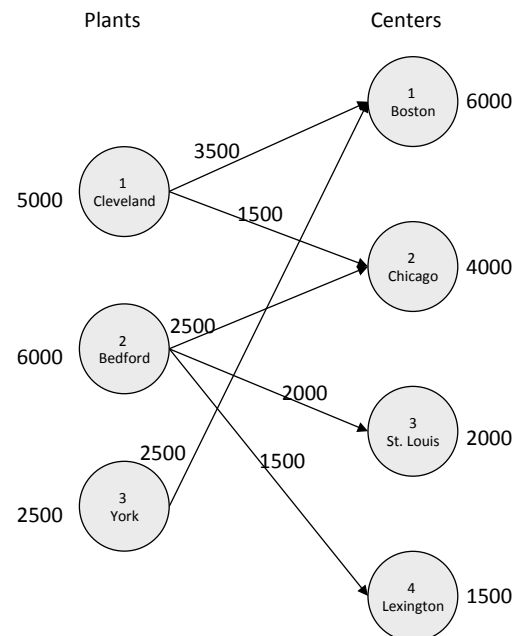
Problem /Users/mike/transport.cmpl
Nr. of variables 12
Nr. of constraints 7
Status optimal
Solver name COIN-OR clip
Objective name costs
Objective value 39500 (min!)

```

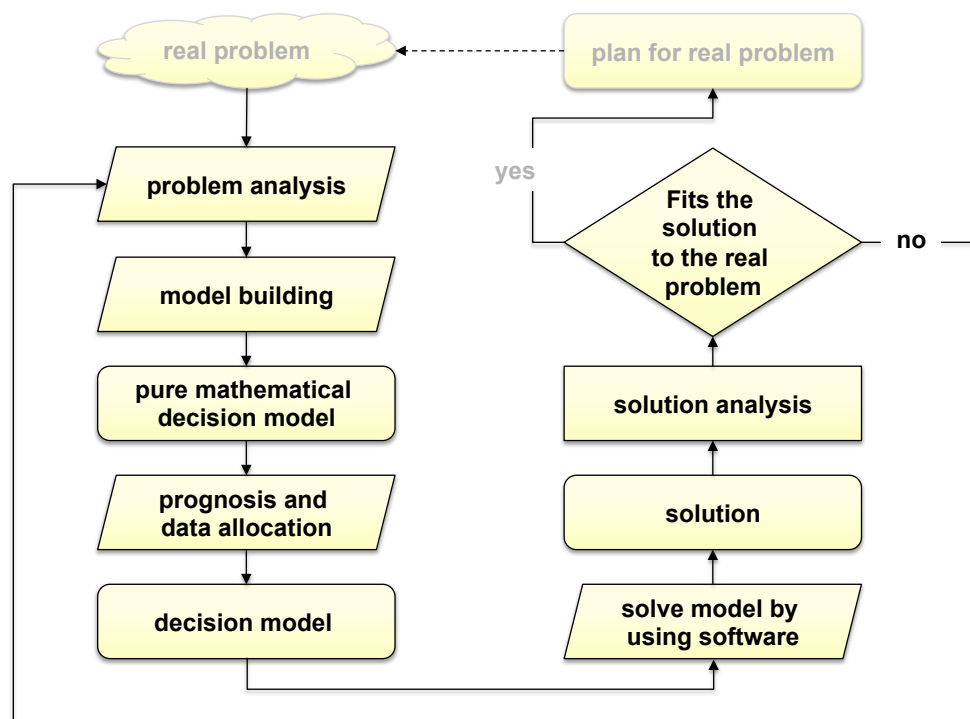
Variables	Name	Type	Activity	Lower bound	Upper bound	Marginal
x(1,1)		C	3500	0	Infinity	0
x(1,2)		C	1500	0	Infinity	0
x(1,3)		C	0	0	Infinity	8
x(1,4)		C	0	0	Infinity	6
x(2,1)		C	0	0	Infinity	1
x(2,2)		C	2500	0	Infinity	0
x(2,3)		C	2000	0	Infinity	0
x(2,4)		C	1500	0	Infinity	0
x(3,1)		C	2500	0	Infinity	0
x(3,2)		C	0	0	Infinity	4
x(3,3)		C	0	0	Infinity	6
x(3,4)		C	0	0	Infinity	6

Constraints	Name	Type	Activity	Lower bound	Upper bound	Marginal
supplies_1		E	5000	5000	5000	1
supplies_2		E	6000	6000	6000	4
supplies_3		E	2500	2500	2500	-
demands_1		E	6000	6000	6000	2
demands_2		E	4000	4000	4000	1
demands_3		E	2000	2000	2000	-2
demands_4		E	1500	1500	1500	-1



## 5 Transportation problems

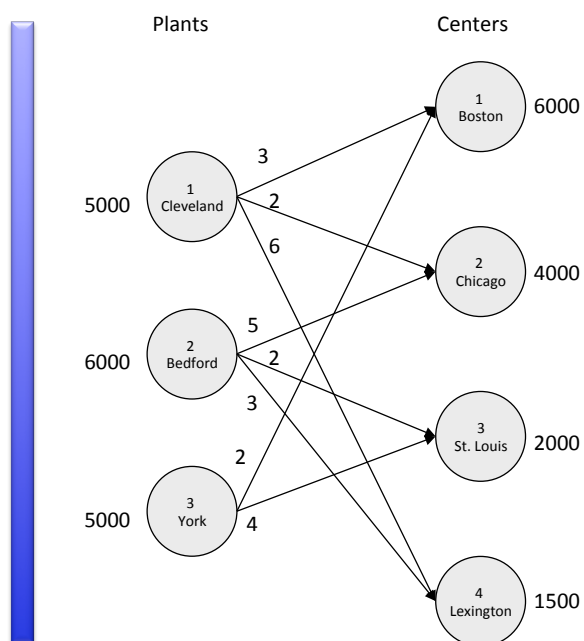


## 4.7 Transportation problems

⇒ Problems and solutions

- The total demand and the total supply are not equal.  $\sum_{i=1}^m s_i \neq \sum_{j=1}^n d_j$ 
  - Change the supply and demand constraints from = constraints to  $\leq$  constraints.
  - If  $\sum_{i=1}^m s_i > \sum_{j=1}^n d_j$  then  $\sum_{j=1}^n x_{ij} \leq s_i \quad ; i = 1(1)m$ .
  - If  $\sum_{i=1}^m s_i < \sum_{j=1}^n d_j$  then  $\sum_{i=1}^m x_{ij} \leq d_j \quad ; j = 1(1)n$ .
- Transport between source  $i$  and destination  $j$  is not possible.
  - Use of a 2-tupel set for the valid combinations of the sources and the destinations or
  - Set the costs  $c_{ij}$  to a big  $M$ .

## 5 Transportation problems



$$\sum_{(i,j) \in routes} c_{ij} \cdot x_{ij} \rightarrow \min!$$

s.t.

$$\sum_{(k,j) \in routes, k=i} x_{kj} \leq s_i \quad ; i = 1(1)m$$

$$\sum_{(i,l) \in routes, l=j} x_{il} = d_j \quad ; j = 1(1)n$$

$$x_{ij} \geq 0 \quad ; (i,j) \in routes$$

## 5 Transportation problems

```

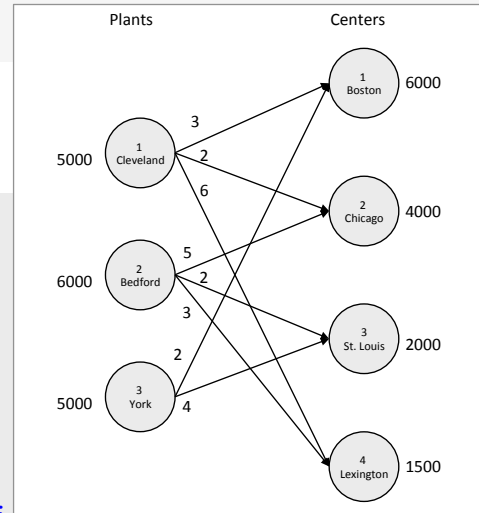
parameters:
    routes := set( [1,1],[1,2],[1,4],
                   [2,2],[2,3],[2,4],
                   [3,1],[3,3] );

    plants := 1(1)3;
    centers := 1(1)4;
    s[plants] := (5000,6000,5000);
    d[centers] := (6000,4000,2000,1500);
    c[routes] := ( 3, 2, 6, 5, 2, 3, 2, 4 );

variables:
    x[routes]: real[0..];

objectives:
    costs: sum{ [i,j] in routes : c[i,j]*x[i,j] } ->min;

constraints:
    supplies {i in plants : sum{j in (routes *> [i,*]): x[i,j]}<= s[i];}
    demands  {j in centers: sum{i in (routes *> [*,j]): x[i,j]}= d[j];}
  
```



## 5 Transportation problems

```

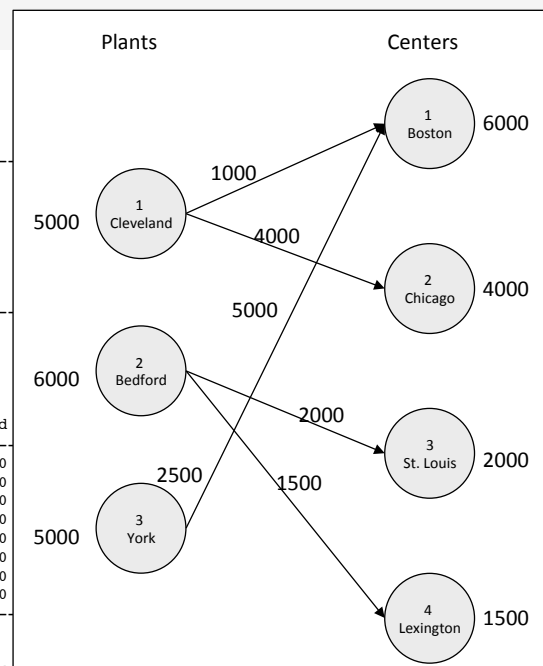
Problem      transportation2.cmpl
Nr. of variables      8
Nr. of constraints    7
Objective name      costs
Solver name         COIN-OR clp
Display variables    (all)
Display constraints  (all)
  
```

```

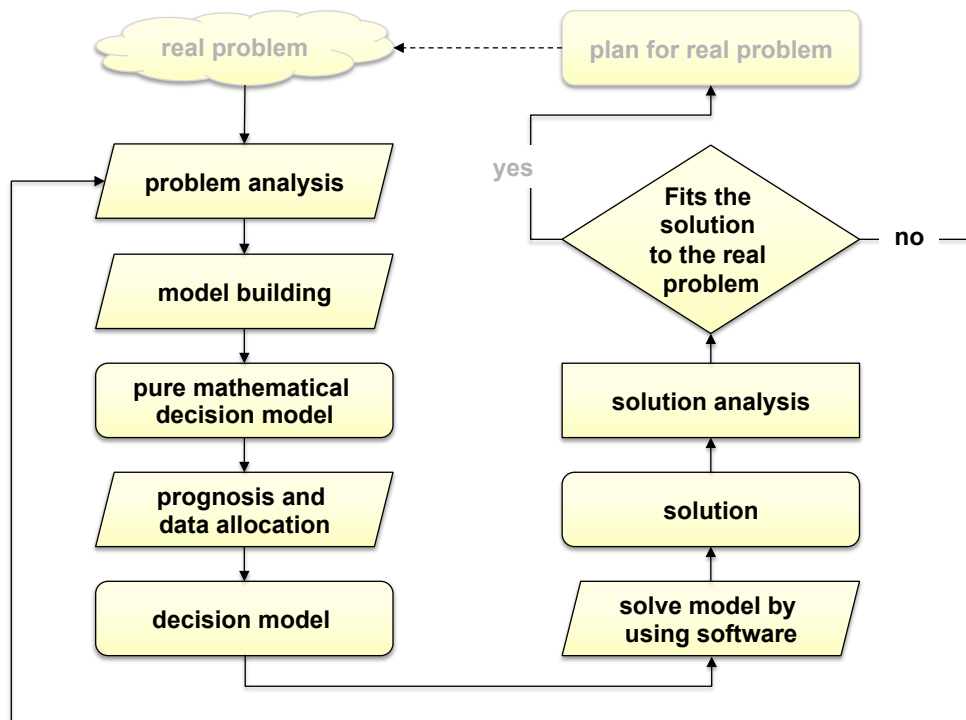
Objective status      optimal
Objective value        29500 (min!)
  
```

Variables	Type	Activity	Lower bound
x[1,1]	C	1000	0
x[1,2]	C	4000	0
x[1,4]	C	0	0
x[2,2]	C	0	0
x[2,3]	C	2000	0
x[2,4]	C	1500	0
x[3,1]	C	5000	0
x[3,3]	C	0	0

Constraints	Type	Activity	Lower bound
supplies_1	L	5000	-Infinity
supplies_2	L	3500	-Infinity
supplies_3	L	5000	-Infinity
demands_1	E	6000	6000
demands_2	E	4000	4000
demands_3	E	2000	2000
demands_4	E	1500	1500



## 5 Transportation problems



## 5 Transportation problems

⇒ It is demanded for technological reasons that the supply from Cleveland (i=1) to Boston (j=1) is either zero, or exactly corresponds to the supply from Cleveland to Chicago (j=2).

$$x_{11} = x_{12} \cdot y \quad : y \in \{0,1\}$$

### ⇒ Product of variables

- A product of variables cannot be a part of an LP or MIP model, because such a variable product is a non-linear term. But if one factor of the product is an integer variable then it is possible to formulate an equivalent transformation using a set of specific linear inequations. [cf. Rogge/Steglich (2007)]
- Let  $w := u \cdot v$  a product of variables and  $v$  integer.
- The following Lemmas distinguish between
  - Lemma 1:  $v$  is a Boolean variable and
  - Lemma 2:  $v$  is an integer.



## 5 Transportation problems

⇒ **Lemma 1:**  $w := u \cdot v, \underline{u} \leq u \leq \bar{u}$  ( $u$  real or integer),  $v \in \{0,1\}$

is equivalent to

$u$  real or integer,  $v \in \{0,1\}$

and

$$(a) \quad \underline{u} \cdot v \leq w \leq \bar{u} \cdot v$$

$$(b) \quad \underline{u} \cdot (1-v) \leq u - w \leq \bar{u} \cdot (1-v).$$

$$x_{11} = x_{12} \cdot y \quad ; \quad y \in \{0,1\}, 0 \leq x_{12} \leq 6000$$

is equivalent to

$$x_{12} \text{ real}, y \in \{0,1\}$$

and

$$(a) \quad x_{11} \leq 6000 \cdot y$$

$$(b) \quad x_{12} - x_{11} \leq 6000 \cdot (1-y)$$

⇒ **Proof:**  $w := u \cdot v$  is equivalent to

$$w = \begin{cases} 0, \underline{u} \leq u \leq \bar{u}, \text{ für } v=0 \\ u, \underline{u} \leq u \leq \bar{u}, \text{ für } v=1. \end{cases}$$

If  $v = 0$  then (a)  $w = 0$

and (b)  $\underline{u} \leq u \leq \bar{u}$ .

If  $v = 1$  then (b)  $w = u$

and (a)  $\underline{u} \leq u \leq \bar{u}$ .

## 5 Transportation problems

```
parameters:
    routes := set( [1,1],[1,2],[1,4],
                   [2,2],[2,3],[2,4],
                   [3,1],[3,3] );
    plants := 1(1)3;
    centers := 1(1)4;
    s[plants] := (5000,6000,5000);
    d[centers] := (6000,4000,2000,1500);
    c[routes] := ( 3, 2, 6, 5, 2, 3, 2, 4 );

variables:
    {i in plants, j in centers:
        x[i,j] :real[0..s[i]];}
    y:binary;

objectives:
    costs: sum{ [i,j] in routes : c[i,j]*x[i,j] } ->min;

constraints:
    supplies {i in plants : sum{j in (routes *> [i,*]): x[i,j]}<= s[i];}
    demands {j in centers: sum{i in (routes *> [*,j]): x[i,j]}= d[j];}
    x[1,1]=x[1,2]*y;
```

## 5 Transportation problems

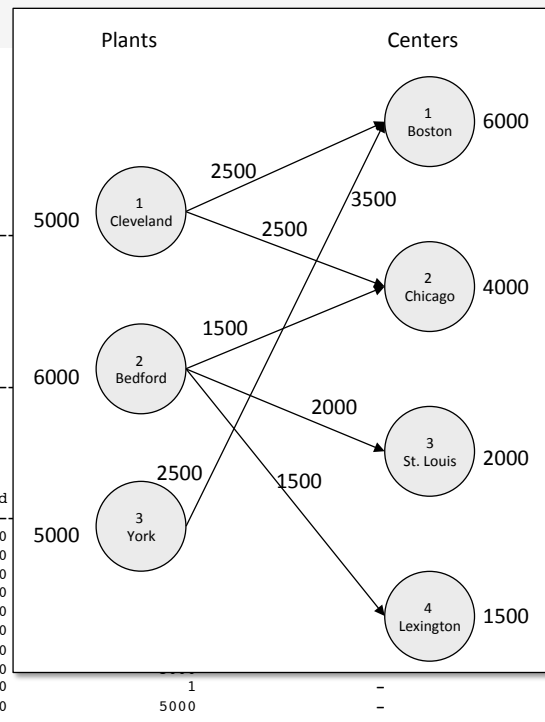
```

-----
Problem                transportation3.cmpl
Nr. of variables        10
Nr. of constraints      11
Objective name          costs
Solver name             COIN-OR cbc
Display variables       (all)
Display constraints     (all)
-----

Objective status        optimal
Objective value         35500 (min!)

Variables
Name                   Type          Activity    Lower bound
-----
x[1,1]                 C              2500         0
x[1,2]                 C              2500         0
x[1,4]                 C               0           0
x[2,2]                 C              1500         0
x[2,3]                 C              2000         0
x[2,4]                 C              1500         0
x[3,1]                 C              3500         0
x[3,3]                 C               0           0
y                      B               1           0
y_x[1,2]               C              2500         0
...

```



## Literature

Anderson/Sweeney/Williams/Martin: An Introduction to Management Science - Quantitative Approaches to Decision Making, 13th ed., South-Western, Cengage Learning 2008.

Domschke, W./Drexl, A.: Einführung in Operations Research, 8. Aufl., Berlin u.a. 2011.

Hillier F. S./Lieberman, G. J.: Introduction to Operations Research, 9th ed., McGraw-Hill Higher Education 2010.

Rogge, R./Steglich, M.: Betriebswirtschaftliche Entscheidungsmodelle zur Verfahrenswahl sowie Auflagen- und Lagerpolitiken, in: Diskussionsbeiträge zu Wirtschaftsinformatik und Operations Research 10/2007, Martin-Luther-Universität Halle-Wittenberg 2007.

Steglich, M./Schleiff, Th.: CMPL - <Coliop|Coin> Mathematical Programming Language – Manual, latest ed.

Suhl, L./Mellouli, T.: Optimierungssysteme, 2. Aufl., Berlin u.a. 2009.

Williams, H.P.: Model Building in Mathematical Programming, 4th ed., John Wiley & Sons.

Winston, W.L.: Operations Research - Applications and Algorithms, 4th ed., Thomson Press, 2003.

**Thank you for your attention.**

**If you have any questions, please feel free to ask.**

**Спасибо за ваше внимание**